Published by Institute of Physics Publishing for SISSA/ISAS
Received: May 4, 2004
Accepted: July 2, 2004

Non-abelian field strength bilocal correlator at finite temperature in the model of dilute instanton gas

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Abstract: Bilocal correlator in gluodynamics is evaluated at finite temperature in the framework of dilute instanton gas model. It is demonstrated that vacuum correlation length decreases with a growing temperature. Comparison with lattice calculations shows that instanton gas model does not describe real vacuum of gluodynamics. Density of instantons and the structure of nonperturbative vacuum of gluodynamics are discussed.

Keywords: Nonperturbative Effects, Solitons Monopoles and Instantons, QCD.

JHEP07(2004)007

Contents

1. Introduction

Nonperturbative fields play important role in the vacuum of QCD and gluodynamics, and determine many properties of the theory, in particular, they are responsible for the confinement. At present there does not exist a commonly accepted picture of nonperturbative vacuum. A well developed and successful in the description of many properties of QCD model is the instanton liquid model, suggested by Shuryak [1] and Diakonov and Petrov [2]. The main nonperturbative fields in this model are rather well separated instantons and antiinstantons, their interaction being not very strong (thus the name "liquid"); instanton density is approximately $1\,\mathrm{fm}^{-4}$. This model solves a number of problems in QCD, in particular it naturally leads to spontaneous chiral symmetry breaking and explains the η' meson mass. However, there is a number of problems of fundamental importance, which cannot be explained in this model — it remains unclear, how to make such instanton-antiinstanton ensemble stable (infrared inflation of instantons), and confinement cannot be explained in the instanton liquid model.

Another model is the model of stochastic vacuum, formulated in the framework of vacuum correlators method, which was developed by Dosch and Simonov [3]. Nonperturbative vacuum of gluodynamics and QCD is parametrized by gauge invariant vacuum averages of gluon field strength tensor — vacuum correlators. Fields are supposed to be stochastic, i.e. the main contribution to different physical quantities comes from the bilocal correlator, while correlators with larger number of fields give only small corrections (the property of gaussian dominance). This model successfully describes many phenomena in QCD (see the review papers [6, 7]). The most important property of this picture of vacuum is that it includes confinement — string tension is expressed through the function $D(z^2)$, which parametrizes bilocal correlator, $\sigma = 0.5 \int d^2z D(z^2)$ [3]. The problem of infrared inflation of instantons is also naturally solved in the stochastic vacuum. First, it was shown in [4] that standard perturbation theory changes in the nonperturbative stochastic vacuum, and contribution of large instantons to physical quantities becomes finite. Next, direct interaction of instanton with nonperturbative vacuum fields was shown [5] to lead to stabilization in instanton size, the average size of instantons being of the order of vacuum correlation length. Distribution of instantons over sizes was found to be in good agreement with lattice data, with a maximum at $\rho_c \simeq 0.25\,\mathrm{fm}$.

Fields are not stochastic in the model of instanton liquid, since instantons give significant contributions not only to the bilocal correlator, but to higher correlators as well. This is clear already from the fact that although it is well known that instantons do not explain confinement, bilocal correlator is nonzero in the instanton gas, and it's contribution to the Wilson loop should be compensated by the contribution of higher correlators [8].

An important property of vacuum is the scaling of string tensions between sources in different representations of color group, which is observed in the lattice calculations [9]. It was shown in [10] that instanton contribution to the force between heavy quarks in different representations violates Casimir scaling, and this leads to a constraint on instanton density.

In the present paper we study instantons' contribution to the bilocal correlator at finite temperature.[1] We demonstrate that correlation length of bilocal correlator, evaluated on calorons (calorons are instanton-like configurations at finite temperature) in the lowest order in caloron density has significant dependence on temperature. At the same time it is found in the lattice calculations that within errors correlation length for chromomagnetic fields does not depend on the temperature in the whole region between zero and critical temperature T_c [13].

2. Chromoelectric and chromomagnetic correlators

Gauge invariant two-point correlators of field strength tensor in gluodynamics are defined as

$$D_{\mu\rho,\nu\sigma} = g^2 \left\langle \mathrm{Tr} \left(F_{\mu\rho}(x)\Phi(x,y)F_{\nu\sigma}(y)\Phi^\dagger(x,y)\right)\right\rangle , \tag{2.1}$$

where $F_{\mu\nu} = F^a_{\mu\nu}t^a$, and

$$\Phi(x,y) = \mathrm{Pexp}\left(igz^\mu \int_0^1 ds\, A_\mu(y+sz)\right) \tag{2.2}$$

is the parallel transporter along the straight line between points x and y; $z = x - y$. Bilocal correlator is defined by two independent functions:

$$\begin{aligned} D_{\mu\rho,\nu\sigma}(z) = {} & (\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\rho\nu})\left(D(z^2) + D_1(z^2)\right) + \\ & + (z_\mu z_\nu \delta_{\rho\sigma} - z_\mu z_\sigma \delta_{\rho\nu} + z_\rho z_\sigma \delta_{\mu\nu} - z_\rho z_\nu \delta_{\mu\sigma})\,\frac{\partial D_1(z^2)}{\partial z^2}\,. \end{aligned} \tag{2.3}$$

[1]Bilocal correlator in the instanton gas at zero temperature was evaluated in [11, 8]. Similar calculations were performed in [12].

At finite temperature (which is defined through periodic or antiperiodic boundary conditions for fields along the time axis $\phi(\vec{x}, t) = \pm\phi(\vec{x}, t+\beta)$, $\beta = 1/T$) it is natural to consider electric and magnetic correlators separately. They are defined by four independent functions $D^{\mathrm{E}}(x^2)$, $D_1^{\mathrm{E}}(x^2)$, $D^{\mathrm{B}}(x^2)$ $D_1^{\mathrm{B}}(x^2)$:

$$\begin{aligned} g^2 \left\langle \mathrm{Tr}\left(E_i(x)\Phi(x,y)E_j(y)\Phi^\dagger(x,y)\right)\right\rangle &= \delta_{ij}\left(D^{\mathrm{E}} + D_1^{\mathrm{E}} + z_4^2 \frac{\partial D_1^{\mathrm{E}}}{\partial z^2}\right) + z_i z_j \frac{\partial D_1^{\mathrm{E}}}{\partial z^2}\,, \\ g^2 \langle \mathrm{Tr}\left(B_i(x)\Phi(x,y)B_j(y)\Phi^\dagger(x,y)\right)\rangle &= \delta_{ij}\left(D^{\mathrm{B}} + D_1^{\mathrm{B}} + \vec{z}^2 \frac{\partial D_1^{\mathrm{B}}}{\partial z^2}\right) - z_i z_j \frac{\partial D_1^{\mathrm{B}}}{\partial z^2}\,. \end{aligned} \tag{2.4}$$

It is obvious, that if the field is selfdual then $D_1^{\mathrm{E}} = D_1^{\mathrm{B}} = 0$, $D^{\mathrm{E}} = D^{\mathrm{B}}$ (see [8]). Thus it is sufficient to consider chromomagnetic correlator only.

The very fact that for instantons $D^{\mathrm{E}} = D^{\mathrm{B}}$ implies that instantons cannot be the only nonperturbative fields in the vacuum of gluodynamics, because it is known from lattice calculations that chromoelectric and chromomagnetic condensates have different dependences on temperature. In particular, at the deconfinement phase transition temperature chromoelectric condensate abruptly vanishes, while chromomagnetic condensate stays intact [13]. Temperature dependence of chromomagnetic bilocal correlator was found analytically in [14], and it was demonstrated that chromomagnetic condensate grows slowly with increasing temperature up to $2T_c$, and grows as $\langle H^2\rangle_T \propto g^8(T)T^4$ at higher temperatures; magnetic correlation length was shown to decrease as $\lambda_m(T) \propto 1/(g^2(T)T)$ at $T \geq 2T_c$.

3. Calorons' contribution to the chromomagnetic correlator

Instanton field with the center at the origin in the singular gauge is given by

$$\begin{aligned} A_\mu^{a\,(\mathrm{inst})}(x) &= -\frac{1}{g}\bar{\eta}^a_{\mu\nu}\partial_\nu \ln \Pi^{(\mathrm{inst})}(x)\,, \\ \Pi^{(\mathrm{inst})}(x) &= 1 + \frac{\rho^2}{x^2} = 1 + \frac{\rho^2}{r^2+\tau^2}\,, \end{aligned} \tag{3.1}$$

where $x = (\vec{r}, \tau)$. This expression is easily generalized to the case of a chain of instantons along the time axis, i.e. caloron [15]:

$$\begin{aligned} A_\mu^a(x) &= -\frac{1}{g}\bar{\eta}^a_{\mu\nu}\partial_\nu \ln \Pi(x)\,, \\ \Pi(x) &= 1 + \sum_{n=-\infty}^{\infty} \frac{\rho^2}{r^2 + (\tau - n\beta)^2} = 1 + \frac{\pi\rho^2}{\beta r}\frac{\mathrm{sh}(2\pi r/\beta)}{\mathrm{ch}(2\pi r/\beta) - \cos(2\pi\tau/\beta)}\,, \end{aligned} \tag{3.2}$$

where $\beta = 1/T$ is the inverse temperature.

Caloron field has the form

$$
\begin{aligned}
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu \,, \\
gB^a_i &= -\delta_{ai} g_1(x) + \varepsilon_{aij} \frac{x_j}{r} g_2(x) + \frac{x_a x_i}{r^2} g_3(x) \,, \\
E^a_i &= B^a_i \,, \\
g_1(x) &= \frac{1}{r}\frac{1}{\Pi}\frac{\partial \Pi}{\partial r} + \frac{1}{\Pi}\frac{\partial^2 \Pi}{\partial \tau^2} - \frac{1}{\Pi^2}\left(\frac{\partial \Pi}{\partial \tau}\right)^2 + \frac{1}{\Pi^2}\left(\frac{\partial \Pi}{\partial r}\right)^2 , \\
g_2(x) &= -\frac{2}{\Pi^2}\frac{\partial \Pi}{\partial r}\frac{\partial \Pi}{\partial \tau} + \frac{1}{\Pi}\frac{\partial^2 \Pi}{\partial r \partial \tau} \,, \\
g_3(x) &= \frac{2}{\Pi^2}\left(\frac{\partial \Pi}{\partial r}\right)^2 - \frac{1}{\Pi}\frac{\partial^2 \Pi}{\partial r^2} + \frac{1}{r}\frac{1}{\Pi}\frac{\partial \Pi}{\partial r} \,.
\end{aligned}
\tag{3.3}
$$

Owing to the selfduality of caloron field $F_{\mu\nu} = \tilde{F}_{\mu\nu}$ chromoelectric field is equal to chromomagnetic one.

Next step is the evaluation of the parallel transporter on caloron:

$$
\begin{aligned}
\Phi(x,y) &= \mathrm{Pexp}\left(igz^\mu \int_0^1 ds\, A_\mu(y+sz)\right) = \mathrm{Pexp}\left(-iz^\mu \bar{\eta}^a_{\mu\nu} t^a \int_0^1 ds\, \partial_\nu \ln \Pi(y+sz)\right) \\
&= \mathrm{Pexp}\left(-it^a \int_0^1 ds\, \left[\bar{\eta}^a_{ij} z_i \partial_j \ln \Pi + \bar{\eta}^a_{i4} z_i \partial_4 \ln \Pi + \bar{\eta}^a_{4j} z_4 \partial_j \ln \Pi\right]\right) , \\
z &= x - y \,.
\end{aligned}
\tag{3.4}
$$

For comparison with lattice results we will consider the case $z_4 = 0$. Then

$$
\Phi(x,y) = \mathrm{Pexp}\left(-it^a \int_0^1 ds\, \left[\bar{\eta}^a_{ij} z_i \partial_j \ln \Pi + \bar{\eta}^a_{i4} z_i \partial_4 \ln \Pi\right]\right) . \tag{3.5}
$$

In general one has to evaluate path ordered integral. However there are special cases, when the exponent commutes at different values of s. Obviously, these cases are $\tau = 0$, $\tau = \beta/2$ and $\tau = \beta$, since $\partial_4 \ln \Pi(r, \tau = 0; \beta/2; \beta) = 0$. Another example is the single instanton field, since $\partial_4 \ln \Pi^{(\mathrm{inst})} = \tau(1/r)(\partial \ln \Pi^{(\mathrm{inst})}/\partial r)$.

To evaluate the contribution of caloron gas to the two-point correlator in the lowest order in density one has to average single-caloron contribution over it's position, or, which is the same, to average over y at a given $z = x - y$.

Averaging over y comes to the integral $\int d^4y = \int d^3\vec{y} \int_0^\beta dy_4$. Averaging over three-dimensional vector $\vec{y}$ does not make any problems, but the integral over y_4 is not tractable, since correlator can be numerically calculated only when $\tau = 0, \beta/2, \beta$. Nevertheless it will be seen from comparison with the single-instanton case that at low temperatures calculations with $\tau = 0$ are sufficient, and at high temperatures correlation lengths for $\tau = 0$ and $\tau = \beta/2$ cases coincide, which makes averaging trivial.

4. Numerical results

Let us use the equation

$$
g^2 \langle \mathrm{Tr}\left(B_i(x)\Phi(x,y)B_j(y)\Phi^\dagger(x,y)\right)\rangle_{\vec{y}} = \delta_{ij}\left(d + d_1 + \vec{z}^2 \frac{\partial d_1}{\partial z^2}\right) - z_i z_j \frac{\partial d_1}{\partial z^2} \,, \tag{4.1}
$$

JHEP07(2004)007

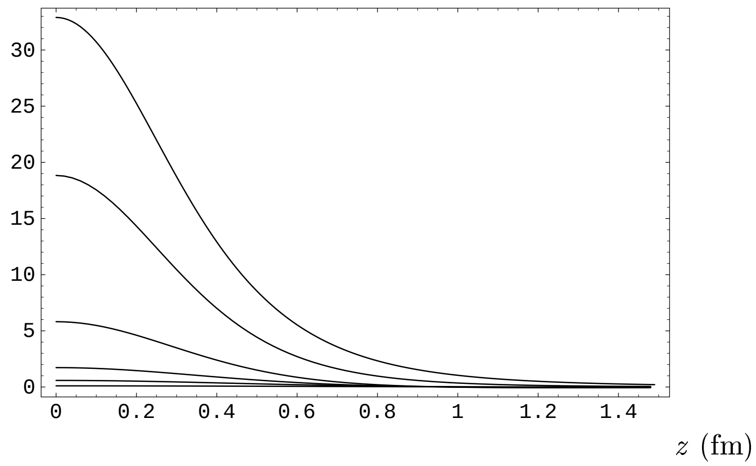

Figure 1: $d^{(\mathrm{inst})}(\tau, z^2)$ at $\tau = (0, 0.15, 0.3, 0.45, 0.6)\,\mathrm{fm}$; $\rho = 0.3\,\mathrm{fm}$. Upper line corresponds to $\tau = 0$.

at $z_4 = 0$ to define the functions d and d_1, which depend on z^2 and $y_4 \equiv \tau$: $d = d(\tau, z^2)$; $d_1 = d_1(\tau, z^2)$. Averaging over y_4 yields

$$\begin{aligned}\int_0^\beta d\tau d(\tau, z^2) &= D^{\mathrm{B}}(z^2)\,,\\ \int_0^\beta d\tau d_1(\tau, z^2) &= D_1^{\mathrm{B}}(z^2) = 0\,,\\ D^{\mathrm{B}}(z^2 = 0) &= \frac{4\pi^2}{3}\,.\end{aligned} \tag{4.2}$$

Last equality follows from the relation $g^2 \int d^3\vec{y} \int_0^\beta dy_4 \left(F^a_{\mu\nu}\right)^2 = 32\pi^2$ (the field is selfdual, and this integral is proportional to the topological charge $\sim \int d^4y F\tilde{F}$).

Let us define correlation length λ_τ as the coefficient in the exponent of the best fit to the function $d(\tau, z^2)$:

$$d(\tau, z^2) \simeq e^{-|z|/\lambda_\tau}\,. \tag{4.3}$$

As discussed above, we will evaluate $d(\tau = 0, z^2)$ and $d(\tau = \beta/2, z^2)$ only. However, this information is sufficient to determine the correlation length of the function $D^{\mathrm{B}}(z^2)$. Indeed, by averaging over τ one can find that normalized to 1 functions $d(\tau = 0, z^2)/d(\tau = 0, z^2 = 0)$ and $D^{\mathrm{B}}(z^2)/D^{\mathrm{B}}(z^2 = 0)$ practically coincide. The fact is that instanton is a well localized field configuration, and it's field decreases very fast when moving away from it's center. This is illustrated in the figure 1. It shows function $d^{(\mathrm{inst})}(\tau, z^2)$, calculated for the single instanton configuration for several values of τ. Profile of the function $D^{\mathrm{B}}(z^2)$ is determined by the function $d(\tau = 0, z^2)$. The same is true for caloron when $\beta \gg \rho$, because in this case instantons forming caloron are well separated. At higher temperatures, when $\beta \le 2\rho$ the functions $d(\tau = 0, z^2)$ and $d(\tau = \beta/2, z^2)$ have almost the same correlation length (see figure 2), which makes averaging over τ trivial (it only affects the amplitude of the functions, which can be found from the condition $D^{\mathrm{B}}(z^2 = 0) = 4\pi^2/3$).

JHEP07(2004)007

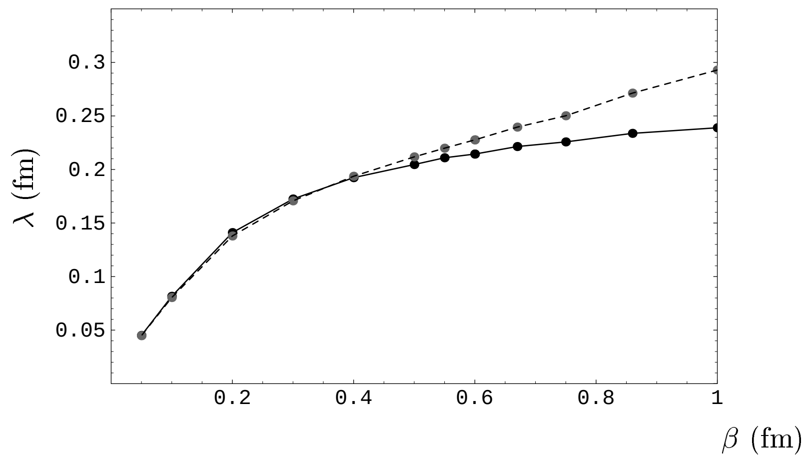

Figure 2: Correlation length of functions $d(\tau = 0, z^2)$ and $d(\tau = \beta/2, z^2)$: $\lambda_{\tau=0}(\beta)$ (solid line) and $\lambda_{\tau=\beta/2}(\beta)$ (dashed line); $\rho = 0.3\,\mathrm{fm}$.

Taking into account all aforesaid, we can restrict our analysis by the function $d(\tau = 0, z^2)$ only. It's correlation length is close to the correlation length of function $D^{\mathrm{B}}(z^2)$ for all values of β.

Chromomagnetic correlator for $\tau = 0, \beta/2, \beta$ takes the form

$$\begin{aligned} g^2 \left\langle \mathrm{Tr}\left(B_i(x)\Phi(x,y)B_j(y)\Phi^\dagger(x,y)\right)\right\rangle_{\vec{y}} &= g^2 \int d^3\vec{y}\left(B_i^a(x)U^{ab}(x,y)B_j^b(y)\right), \\ U^{ab}(x,y) &= \mathrm{Tr}\left(t_a\Phi(x,y)t_b\Phi^\dagger(x,y)\right) \\ &= \frac{1}{2}\delta^{ab}\cos(2\phi) - \frac{1}{2}\varepsilon^{abc}n_c\sin(2\phi) + n_a n_b \sin^2(\phi), \\ n_a\phi &= \frac{1}{2}\bar{\eta}^a_{ij} z_i y_j \int_0^1 ds \left(\frac{1}{r}\frac{\partial \ln \Pi(y+sz)}{\partial r}\right). \end{aligned} \tag{4.4}$$

Correlator defined in (4.4) is presented in figure 3 for several values of β. Corresponding correlation lengths are shown in figure 4.

5. Discussion

Bilocal correlator at finite temperature has been studied on lattice in the paper [13], and it was found that bilocal chromomagnetic correlator practically does not depend on temperature in the whole region from zero to T_c (this dependence is also very weak above critical temperature). At the same time, as it is seen from figure 4, the correlation length in the dilute instanton gas changes by approximately 20% when temperature changes from zero to 300 MeV. Thus, one comes to the conclusion that instanton gas cannot be a correct model for the vacuum of gluodynamics. There are several possible explanations of these lattice data.

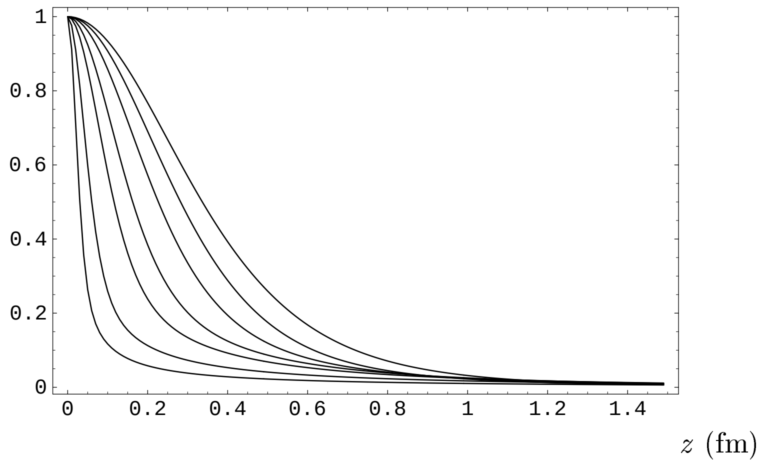

Figure 3: $d(\tau = 0, z^2)/d(\tau = 0, z^2 = 0)$ for $\beta = (0.05, 0.1, 0.2, 0.3, 0.5, 0.86, \infty)\,\mathrm{fm}^{-1}$; $\rho = 0.3\,\mathrm{fm}$. The largest correlation length (upper line) corresponds to zero temperature ($\beta = \infty$), i.e. to instantons' contribution.

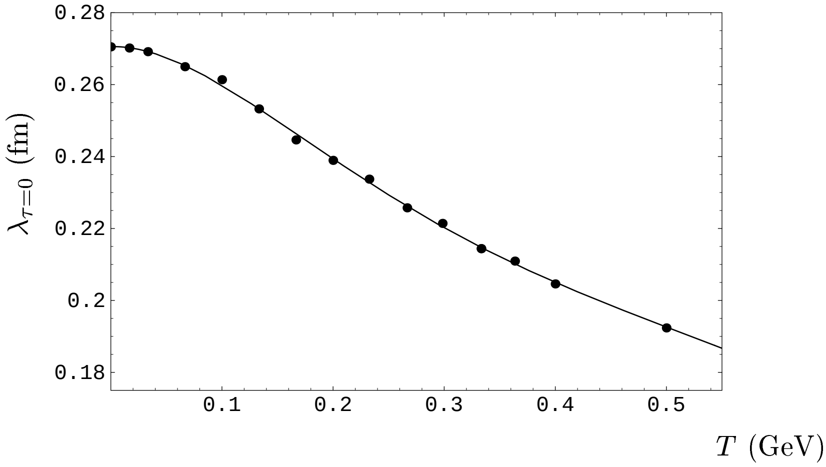

Figure 4: Correlation length as a function of temperature $\lambda_{\tau=0}(T)$; $\rho = 0.3\,\mathrm{fm}$.

1. Instanton density is much less than $1\,\mathrm{fm}^{-4}$, and the main contribution to the correlator comes from other nonperturbative fields, which are responsible for the confinement. This picture agrees well with the model of stochastic vacuum.

2. Instanton density is high enough to make next terms in density expansion change the result qualitatively, leading to the independent of temperature correlation length.

3. Instanton ensemble is rearranged at nonzero temperature, instantons and antiinstantons form molecules, and the contribution of these instanton-antiinstanton molecules to bilocal correlator is significantly different from that of instanton gas. This binding may be connected to the deconfinement phase transition. For example, this is the case for 3-d adjoint Higgs model, where it was found that in the deconfined phase instantons and antiinstantons are bound into molecules [16]. Formation of instanton-

antiinstanton molecules in QCD vacuum and it's connection to the chiral symmetry restoration phase transition is discussed in review [17].

4. It should be noted, that in the last years a different scenario of the caloron vacuum in the confined phase ($T < T_c$) is discussed. There exist field configurations with nontrivial holonomy, the so called KvBLL solution [18]. This scenario [19] suggests that calorons may be composites of dyons and antidyons. Lattice calculations, giving numerical evidence for the existence of these field configurations for the cases of SU(2) and SU(3) groups, are presented in papers [20, 21]. Clearly, in the case of this scenario correlation length in the nonperturbative vacuum has a more complicated temperature dependence, than in the case of caloron gas studied in this paper.

In principle, one can consider one possibility more, when instanton size distribution depends on temperature in such a way, that the resulting correlation length does not depend on T. However, this is ruled out by existing lattice calculations, which demonstrate that within errors instanton size distribution stays the same in the temperature region from 0 to T_c [22].

In this paper we have found the contribution of instanton gas to the bilocal correlator at finite temperature. Comparison of the temperature dependence of correlation length with lattice results allows to make the conclusion that if instanton density is not too high and corrections to the dilute gas approximation are not significant, then instanton density should be much less than $1\,\mathrm{fm}^{-4}$, the value standard for instanton liquid model. Vacuum of non-abelian gauge theory at finite temperature is a very complicated system, and it is natural to expect that it can be a combination of the above listed possibilities.

Acknowledgments

Authors are grateful to Yu.A. Simonov for useful comments. This work is supported by NSh-1774.2003.2 grant and by Federal Program of the Russian Ministry of Industry, Science and Technology No 40.052.1.1.1112.

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